Exercise 1. **HW3** Prove that if $M$ is a connected manifold, then it is path connected, i.e. any two points in $M$ can be joint by a continuous curve.

Exercise 2. **HW3** Prove that if $M$ is a connected manifold of dimension 1, then it is topologically either $S^1$ or $(0, 1)$.

Exercise 3. **HW3** Let $F : \mathbb{R}^{n+1} \to \mathbb{R}$ be a smooth map and $c \in \mathbb{R}$, such that $DF_a$ is surjective at each $a \in X = F^{-1}(c)$. Suppose the origin $0 \notin X$. Show that the map

$$f : X \to S^n, \quad x \mapsto \frac{x}{||x||}$$

is smooth.

Exercise 4. **HW3** Let $p : \mathbb{R}^3 \to \mathbb{R}$ be the linear projection onto the last coordinate, hence $p$ is a submersion. Consider the restriction of $F = p|_M$ to the subset

$$M = \{(x, y, t) \in \mathbb{R}^3 | xy + t(x^2 + y^2) = 0\}.$$

Decide which level set $F^{-1}(t)$ is a submanifold of the $xy$-plane

Exercise 5. **HW3** Let $p : \mathbb{R}^5 \to \mathbb{R}^3$ be the linear projection onto the last 3 coordinates, hence $p$ is again a submersion. Consider $F = p|_M$ to the subset

$$M = \{(x, y, a_0, a_1, a_2) \in \mathbb{R}^5 | a_0 xy + a_1 x^2 + a_2 y^2 = 0, \quad x, y \text{ not both zero}\}.$$

Show that $M$ is a submanifold of $\mathbb{R}^5$. Decide which level set $F^{-1}(a_0, a_1, a_2)$ is a submanifold of the $xy$-plane. Where does $DF_x$ fail to be surjective?