Math 22a, Introduction to Linear Algebra
Fall 2022

Mode: In person
Venue: Goldsmith Rm TBA (in person)
Time: MW2:30-3:50+Th2:20-3:10
Instructor: Bong Lian (lian@–, Goldsmith 314, X6-3069)
Office Hours: MW12:30-1:15, by appointment
Teaching assistant: TBA

I will use the LATTE forum to communicate with you occasionally. You may post comments or questions that you think will be of general interest to the whole class.

Course Description

In order to sign up for this course, a student should take the Math 22a online placement test at

http://www.brandeis.edu/departments/mathematics/undergraduate/math22placement.html

You should then request an override on Workdays. If qualified, you would be provided an override.

This course will be taught in person.
Textbook: Linear Algebra, by Bong H. Lian, will be posted in installments in the form of lecture notes. They are available for free download (subject to the author’s expressed permission) at
http://people.brandeis.edu/~lian/Math22a/22a-F22.html

Topics we plan to cover include (but not necessarily in this order)

Numbers, sets, and maps
From linear equations, to vectors, to model spaces, to linear maps
Linear algebra of $\mathbb{R}^n$ and solutions to linear equations
Row reduction (Gaussian elimination)
Abstract vector spaces and linear maps
Dimension theory and bases
Bases and orthogonality
Determinants
Eigenvalue problems
*Jordan canonical forms, if time permits

Linear algebra is fundamentally a study of linear equations, but ultimately about linear spaces and their relationships. We will start from counting numbers, leading us to the need of solving linear equations. We will learn about the language and techniques that are necessary for working with linear equations. The language of sets of maps will be fundamental to all this.

Learning Goals
Students in Math 22a will work with vectors and matrices, and their abstract generalizations, use them to solve linear equations, describe the space of solutions, including its bases and dimension; they will study linear maps to analyze vector spaces, their subspaces and quotients; they will study the determinants, eigenvalues and eigenvectors of linear maps.

4-Credit Course

Success in this 4-credit hour course is based on the expectation that students will spend a minimum of 10 hours of study time per week in preparation for class (reading, homework, in-class discussions preparation for exams, etc.).

Grading and Homework

All homework assignments will be posted on the course page.

Each mid-term test will be done during class. Students who miss a test (or exam) will not be granted a make-up test (or exam) unless there is a documented medical or other emergencies. Almost all homework problems will be drawn from the textbook.

Grades will be based on homework, two 1-hour tests (in class), and a final exam (scheduled by the registrar), weighted as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight</th>
<th>Due/Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework</td>
<td>30%</td>
<td>10 sets usually due on Wed</td>
</tr>
<tr>
<td>Tests</td>
<td>15%</td>
<td>Wed Sept 22 and Wed Oct 27</td>
</tr>
<tr>
<td>Classroom participation</td>
<td>5%</td>
<td>every class</td>
</tr>
<tr>
<td>Final Exam</td>
<td>35%</td>
<td>date &amp; time TBA</td>
</tr>
</tbody>
</table>

Students should make sure they do not make commitments that conflict with the test dates.

Policies

You may discuss the homework problems with your classmates; however, if you do, you should write on your homework submission the names of the classmates with whom you have discussed the assignment. You do not need to mention any help you have received from the instructor or the TA. You may not copy the written work of another student or from any other sources, or allow another student to copy your written work. What you submit should be your own work.

You should state the source of a mathematical fact you use when writing up your work, unless the fact you use is something you had learned earlier as part of your prerequisite for 22a. You can state the source by citing a theorem in the textbook, the page number of an exercise we have gone over in class or in a prior homework, or a fact we have proved in class.

Both the instructor and the TA are available during their weekly office hours or by appointments. Students are encouraged to seek help from them on any course related matter.
If you are a student with a documented disability on record at Brandeis University and wish to have a reasonable accommodation made for you in this class, please see me immediately.

Advice

I recommend the following strategy.

(1) Try to prepare before each class. That means you should read (carefully!) the sections to be covered before coming to class, whenever possible. Try the exercises in the book even though you may not be able to get them all. Having thought through the material by yourself makes it a lot easier for you to understand the lectures and ask questions in class.

It is also a very economical way to learn mathematics. For every hour you spend preparing before class, the pay-off could easily be a saving of two to three hours after class.

(2) Keep a note book of mathematical vocabulary. Unlike common English, mathematical English has very few words, but they are very specific in their use. You will learn no more than a 100 new words in 22a. It is very important to master their use. Keeping a note book for these words and how to use them is very important.
Frequently Asked Questions

1. What’s the difference between Maths 22a and 15a?
   • They cover essentially the same syllabus, but do so with very different emphases. Here is a caricature of 22a. Our emphasis in 22a will be roughly 2/3 proofs and 1/3 computational techniques, while the primary emphasis in 15a is techniques. In 22a, computational techniques are more of a means (but an important one!) than an end – computations are exercises more often used in two different ways – to motivate certain concepts, and to confirm your conceptual understanding of certain abstract theorems.
   • Since we move through the textbook at greater pace in 22a, some details will be left out of the lectures. Students are therefore expected to study those details on their own, and are encouraged to ask questions. While some of the problems students see in homework assignments and tests in 22a can be solved directly using techniques they have seen in class, other problems will require greater efforts and more creativity on the students’ part.

2. Is every item on the Math 22a Placement Exam a pre-requisite for 22a?
   Technically, no. However, the test has been a fairly reliable indicator of a student’s readiness to learn math at the pace and level of abstraction that 22a requires. We recognize the diversity of our students’ math background, which means that one person may be well-prepared on some topics but not others, while another person may be just the opposite. The important point here is that to be ready for 22a, a student should be well-prepared on enough topics so they can be ready to learn at the level that 22a requires. Thus, while one student may not have seen anything on 3-space (which is technically not a pre-requisite), but they have done well on everything else on the test, then there is a good chance they will be ready for 22a. By the same token, if a student is quite familiar with the 3-space topics but have forgotten a few things about trigonometry (which is a pre-requisite), they can still do well in 22a because they can then be more at ease with the 3-space topics while trying to relearn things about trigonometry at the same time. But if a student have not seen anything on 3-space and they have also forgotten their trigonometry, then they are much likelier to struggle in 22a.

3. How much time do I need to spend to work on this course?
   It depends on your background and on what you do before and after class. Outside of class lectures, 6-12 hours a week would be near optimal if you spend them wisely. Spending an hour or two thinking about a challenging homework problem before getting any clue is not unusual.

4. What does it mean by "spend them wisely"?
   Here are a few pointers:
   • spend 1-2 hours doing critical reading, including exercises, of material to be covered in the next lecture; pinpoint what you don’t understand and where you are stuck;
• pay special attention to and on those points during the lecture, while confirming that you really understand what you think you understand from your reading; ask questions during lectures to clarify the points you missed;
• work on the assigned homework problems as soon as possible; working in a group is encouraged;
• do not let your homeworks lapse; if the lectures are beginning to seem too fuzzy to you to help you do your homework, and you are too afraid to ask questions in class, you have probably waited too long; do not let problems linger – you can fall behind much faster than you might think;
• make full use of my office hours and/or appointments with me; to make an appointment with me, all you need is to send me an e-mail; you can do the same for getting the TA’s help.

5. Concepts in abstract math often sound fuzzy to me; how do I overcome that in 22a?
• Abstract concepts and theorems quite often seem opaque initially, and some of them look and sound similar; while the lectures help, you also need well-disciplined and critical thinking to clear up confusion; developing this skill takes time and patience;
• in practice, one helpful way to differentiate closely related concepts and theorems is to associate them to simple pictures and examples.

6. The textbook doesn’t have answers to numerical exercises. How do I make sure that my answers are correct before handed them in?
• You are encouraged to work in a group, and compare numerical answers with your classmates. If your answer is different, you should double check that you have done your work correctly.