TSINGHUA MATHCAMP 2014 COURSE: LINEAR ALGEBRA COURSE DESCRIPTION

BONG LIAN, INSTRUCTOR

Linear algebra may be thought of as the study of linear equations, like the familiar equation $x + y + z = 1$. But the subject permeates other ends of the mathematical world including topology, geometry, analysis, not to mention algebra itself! As a tool, it's power can also be felt throughout mathematical sciences at large, reaching the deepest corners of physics, biology, finance and economics.

We, however, will start – from the very beginning – with a brief introduction to linear equations and review basic computational techniques for solving them. We then introduce Euclidean geometry into their study, which brings us to the fundamental notions of vectors and linear spaces, allowing us to begin to talk about the shapes and sizes of solutions to linear equations. We then learn about important notions including bases, linear dependence, dimension, orthogonality, and linear transformations. That would allow us to eventually understand what it truly means to say that "the equation $x + y + z = 1$ describes a plane in Euclidean 3-space," or to answer questions like "given a point P and a plane H in 3-space, how do we find the point in H closest to P?" or "given n points on a plane, can we find a straight line that is the closest to all those points?" Along the way, we shall see other applications such as Markov processes, least square problems and Lagrange interpolations. We shall then move on to more advanced topics such as determinants and eigenvalue problems.

Specific topics we plan to discuss in the 14 lectures include (* means "if time permits")

Equations
Linear systems
Gaussian reduction
Linear Euclidean geometry
Vectors, matrices and their operations
Bases and orthogonality
Determinants*
Eigenvalue problems*
Abstract vector spaces and linear maps
Invariant spaces and Jordan canonical forms*

We will use the book "*Linear Algebra*" that I wrote. Students can download chapters of it at
`http://people.brandeis.edu/~lian/TsinghuaMathcamp2014/TsinghuaMathcamp2014.html`

As for pre-requisite or background, Mathcampers are not expected to have had any prior advanced training in mathematics, but good working knowledge of high school algebra – equations in one or two variables, real numbers – some familiarity with calculus and complex numbers, plus a bit curiosity would give them a great start in this couse!

Homework problems will be assigned in class almost daily during the lecture. A research project based on linear algebra (and a little bit of calculus) will be announced during the first week of Mathcamp. Mathcampers will be meeting and working with two Coaches in this course, in addition to the Instructor. We are happy to have Brian Chung from MIT and Yifan Li from Tsinghua University to be our Coaches. The Coaches will be meeting with Mathcampers on a daily basis, to work with them on their homework assignments and their research project. Mathcampers are also very much encouraged to work with each other.