Tsinghua Mathcamp Summer 2014

Topological Dynamics

Dmitry Kleinbock
Brandeis University

The title of the course consists of two ingredients: "topology" and "dynamics", which come from the Greek words $\tau o \pi o \varsigma$ (place) and $\delta \upsilon \nu \alpha \mu \iota \kappa o \varsigma$ (powerful $\to$ energetic $\to$ active). The message seems to be clear: this branch of mathematics studies **motion**, defined by some prescribed rule, inside a **space** with some geometric structure. For that one needs:

- a space X, which will be a *topological*, or, in most cases throughout the course, a *metric* space – for example, a interval on the real line or a region in the plane

and

- a map $T : X \to X$, which makes it possible for points of X to move around in X.

The goal is to study trajectories

$$\{x, T(x), T\big(T(x)\big), \dots, T^n(x), \dots\}$$

and answer various questions about their behavior.

The pair (X, T) is called a *dynamical system*. It turns out that many mathematical questions can be phrased in the language of properties of some dynamical systems. Thus, besides being an interesting branch of math in itself, the theory of dynamical systems has numerous connections with other branches – in particular, combinatorics and number theory.

Our course will be based on unpublished lecture notes by an outstanding topological dynamicist, Bryna Kra (Northwestern University, USA). Here is the link to the first eight chapters of her book, substantial part of which I hope to cover:

`http://people.brandeis.edu/~kleinboc/China/kra_lectures.pdf`

The course will in fact start with

`http://people.brandeis.edu/~kleinboc/China/kra_appendices.pdf`

where some background information is reviewed; then, after we become comfortable with notions of abstract topological and metric spaces, we will proceed with the main body of the book, and will try to prove some nice theorems about topological dynamical systems and their applications.

The book (and our course) assumes no pre-requisites except for high-school algebra and geometry, and some familiarity with calculus (such as the notions of limit and continuity). Homework problems will be assigned in class, and some of them will have the potential to be expanded to research projects. Mathcampers will be meeting and working with two Coaches in this course, in addition to the Instructor. We are happy to have Yiting Li from Brandeis Univeristy and Huajin Zhang from Tsinghua University to be our Coaches. The Coaches will be meeting with Mathcampers on a daily basis, to work with them on their homework assignments and their research project, Mathcampers are also very much encouraged to work with each other.

See you next month!