Review Sheet for Math 5a Final Exam

- The Math 5a final exam will be **Tuesday, May 1 from 9:15 am – 12:15 p.m.**
- **Location:** Gerstenzang 123
- The final exam is **cumulative** (i.e., it will cover all the material we covered in class this semester).

However, the material covered since the last exam (Section 2.7, Modeling with Functions, Sections 4.1–4.5 and Trigonometry) will make up **at least 60%** of the exam:

- Section 2.7: One-to-one Functions and their Inverses
- Modeling with Functions
- Section 4.1: Exponential Functions
- Section 4.2: The Natural Exponential Function
- Section 4.3: Logarithmic Functions
- Section 4.4: Laws of Logarithms
- Section 4.5: Exponential and Logarithmic Equations (and Inequalities)
- Trigonometry:
  - * Section 6.1: Angle Measure
  - * Section 6.2: Trigonometry of Right Triangles
  - * Trig Ratios in a Circle, Families of Angles, Unit Circle, Trig Equations and Trig Graphs

- **You must bring your student ID to the final exam.**
- **On the exam, you will be asked to show all your work.**

  We are grading your ability to clearly communicate your understanding of *how* to solve each problem much more than your ability to get the correct answer. In fact, a correct answer with little or no work might not receive any points at all.

  Showing your work clearly also gives us justification for assigning partial credit.

- **No calculators will be allowed on the exam.**

  Our exams are closed book: no notes, books, calculators, cell phones or internet-connected devices will be allowed.

- The **solutions** to the review sheet are available on LATTE.
Study Plan:

1. Read through this whole study plan. Check your calendar and set aside enough time to study. **A little time studying every day is MUCH better than a big chunk of time the day before the exam.** Give yourself time to correct mistakes and do more practice problems on anything you don’t get right the first time through.

2. Start by sitting down and taking **Practice Exam 1** (below). Give yourself two hours without interruption to take it (like the actual exam), and do not use any resources (notes, books, calculators, internet, friends), just like the real test.

3. When you’re done, go back and check your answers. For anything you missed, review your notes, reading assignments, homework, quizzes and self-quizzes and try more problems like those.

   Make sure you understand anything you got wrong. Then do enough similar problems until that kind of problem becomes easy for you.

4. Make sure you understand important definitions, formulas, rules and statements of any theorems.

5. Go to office hours or evening help or talk to friends if you have questions about anything.

6. Repeat this whole process for **Practice Exam 2** (below).

7. Finally, repeat this whole process for the **Practice Problems** (below).

8. Go to your **review session** to get any last questions answered, and to find out what questions other students are asking.

9. Hopefully by this point you’ll be feeling confident and ready for the exam!

**Note:** Make sure you really understand what you’re doing. You will probably see problems on the exam that are asked in a different way from problems you have seen before, but if you understand what you’re doing, you should be able to figure them out.
Practice Exam 1 (Problems 1 - 19)

1. Simplify: \( \frac{(a^2b^{-1}c)^{-3}}{abc} \)

2. Solve the following equations:
   (a) \( x^2 - x - 1 = 0 \)  
   (b) \( \frac{1}{1 + x} + \frac{1}{x} = 2 \)  
   (c) \( \frac{2-x}{x^2 - 4} = 0 \)

3. Evaluate the following. If an expression doesn’t exist, just say so.
   (a) \( \ln e^{-2} \)  
   (b) \( \ln \frac{\sqrt{e}}{e} \)  
   (c) \( \log_2 1 \)  
   (d) \( \log 0 \)

4. Suppose Janet bought an iPhone last year for \$650. She found out that if she wanted to sell it now, she could sell it for \$360. To determine how much it might be worth in the future, you decide to model the current sale value for her with a linear function so that the value, \( V \), of the iPhone is a function of time, with time \( t = 0 \) corresponding to when she bought the phone.
   (a) Find a linear equation expressing the value of the iPhone as a function of time (in years).
   (b) Sketch a graph of your linear function from part 4a.
   (c) What does the slope of your line represent? (Explain in words.)

5. Find the inverse function of \( f(x) = 3x^3 + 7 \).

6. Let \( h(x) = \cos(x^2 - 3x) \). Find two functions \( f(x) \) and \( g(x) \) such that \( h(x) = (f \circ g)(x) \).
   **Note:** You may not use the functions \( f(x) = x \) or \( g(x) = x \).

7. Solve the following equations:
   (a) \( 2000 = 10e^{4k} \)  
   (b) \( \ln(x + 1) + \ln(x - 2) = \ln 4 \)

8. Let \( f(x) = \frac{x}{x - 1} \). Find \( \frac{f(x + h) - f(x)}{h} \) and simplify as much as possible.

9. Evaluate the following. If an expression doesn’t exist, just say so. All angle measures are in radians.
(a) \( \sin \frac{-2\pi}{3} \)  
(b) \( \tan \frac{3\pi}{2} \)  
(c) \( \csc \frac{11\pi}{6} \)  
(d) \( \cos \frac{15\pi}{4} \)

10. In each of the following, find the values \( \theta \) in \([0, 2\pi]\) that make the equation true.

(a) \( \sqrt{3} \tan \theta + 1 = 0 \)  
(b) \( 4 \cos^2 \theta - 3 = 0 \)  
(c) \( \theta^2 \sin \theta - \sin \theta = 0 \)

11. Make an accurate sketch of the function

\[
f(x) = \begin{cases} 
\frac{1}{x} & x < -2 \\
(x + 1)^3 - 1 & -2 \leq x < 0 \\
-\cos x + 2 & x \geq 0 
\end{cases}
\]

on the axes below.

**Note:** Your graph must extend to both ends of the \( x \)-axis.

12. Find all angles in the interval \([0, 2\pi]\) that are in the \( \theta = \frac{\pi}{7} \) family.

(Of course, \( \frac{\pi}{7} \) is not one of our special angles.)

13. Suppose you are on a city committee tasked with designing a new park. The area of the rectangular park will be 1,000m\(^2\). Your committee decides to have a brick wall on one side of the park that will cost $90/m. The other three sides will have a low stone wall that costs $30/m. Express the total cost of building the surrounding walls as a function of the length of the brick wall.
14. Find the domain of the function \( g(x) = \sqrt{\frac{x^2 - 16}{x - 2}} \). Write your answer in interval notation.

15. Let \( f(x) \) be the function graphed below:

(a) Find the domain of \( f(x) \).
(b) Find the range of \( f(x) \).
(c) For what values, if any, is \( f(x) = 1? \)
(d) Find \( (f \circ f)(2) \).
(e) On what intervals is \( f(x) \) increasing?
(f) On what intervals is \( f(x) \) positive?
(g) Is \( f(x) \) one-to-one? Why or why not?

16. Solve the following inequality. Write your answer in interval notation.

\[(\ln x + 1)(\ln x - 2) > 0\]

17. Consider the equation

\[\sin^2 \theta - \cos^2 \theta = 0.\]

Solve this equation (i.e., find all values of \( \theta \) in the interval \([0, 2\pi]\) that make the equation true) by using one of the two methods below: (you can choose whichever method you prefer)

- Factor the left hand side and then solve, OR
• Solve by using a trig identity to write the above equation in terms of \( \cos \theta \).

18. Make an accurate sketch of the function

\[
f(x) = \begin{cases} 
\ln(x - 1) + 1 & x > 1 \\
-e^{-x} & x \leq 1
\end{cases}
\]

**Note:** You must label (with both \( x \)- and \( y \)-coordinates) at least 2 points on each piece of the graph.

19. Determine whether each of the following two statements is true or false.

**You must justify your answer.**

**Note:** In mathematics, “true” means that the statement must always be true. “False” means that the statement may sometime be false.

(a) True or False: The domain of \( f(x) = \ln(\ln x) \) is \((1, \infty)\).

(b) True or False: Let \( p(x) = 0 \) be a quadratic equation. If the discriminant of \( p(x) \) is equal to 1, then there is exactly one solution to the equation above.
20. (6 points) Convert to exponential form and simplify: \(
\frac{\sqrt{x y^2 z^3}}{\sqrt[3]{x^5}}
\)

21. (11 points) Solve the following equations:
   
   (a) \( \frac{1}{x^2} + \frac{1}{x} = 3 \)  
   (b) \( e^{3x} + 7 = 0 \)  
   (c) \( (\ln x - 2)(\ln x + 7) = 0 \)

22. (8 points) Evaluate the following. If an expression doesn’t exist, just say so.
   
   (a) \( \ln 0 \)  
   (b) \( \ln \frac{e}{\sqrt{e}} \)  
   (c) \( \log_5 1 \)  
   (d) \( \log_{10} 1 \)

23. (8 points) Sketch the graph of the following function on the axes below. Make sure you clearly label at least 2 points (with \(x\)- and \(y\)-coordinates) on each piece of the graph (so 4 points total).
   
   \( f(x) = \begin{cases} 
   2^{-x} & \text{if } x \leq 0 \\
   -\ln x & \text{if } 0 < x 
   \end{cases} \)

Show all your work!

OVER →
24. (5 points) Verify that \( f(x) = \frac{3x + 2}{3x - 1} \) and \( g(x) = \frac{x + 2}{3x - 3} \) are inverses. (Show all work.)

25. (6 points) A model for the number of people \( N \) in a college community who have heard a certain rumour is given by \( N = P(1 - e^{-0.15d}) \) where \( P \) is the total population of the community and \( d \) is the number of days elapsed since the rumour began. In a community of 1000 students, how long will it take until half of the students have heard the rumour?

Note: You won’t be able to find a number – you should solve until you get an expression that you would plug into your calculator to get a number.

26. (8 points) Let \( f(x) = \frac{3}{x^2} \). Find \( \frac{f(x + h) - f(x)}{h} \) and simplify as much as possible.

27. (8 points) Evaluate the following expressions. If the expression does not exist, just say so. All angle measures are in radians.

(a) \( \cos(\frac{5\pi}{6}) \)  
(b) \( \tan(\frac{3\pi}{2}) \)  
(c) \( \sin(7\pi) \)  
(d) \( \csc(-\frac{4\pi}{3}) \)

28. (10 points) Solve the following equations.

(a) \( x^3 = 16x \)  
(b) \( \log x + \log(x - 2) = \log(x + 4) \)

29. (5 points) Find the domain of the function \( f(x) = \frac{1}{\ln(x)} \). Write your answer in interval notation.

30. (15 points) Solve the following inequalities. Write your answers in interval notation.

(a) \( \frac{x^2 - 9}{x^2 + x - 6} \geq 0 \)  
(b) \( (\ln x + 1)(\ln x - 1) < 0 \)

31. (8 points) The secant line to the function \( f(\theta) = \cos(\theta) \) between 0 and \( \pi \) is the line going through the points \((0,1)\) and \((\pi,-1)\).

(a) Find an equation for this line.

(b) Find an equation for the line perpendicular to the line you found in part 31a and going through the point \((\pi/2,0)\)

Show all your work!
32. (11 points) Consider the given function $h(x) = 2\sin^2(x)$.

(a) Find functions $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$. Note: you may not use $f(x) = x$ or $g(x) = x$ in your answer.
(b) Solve $h(x) = 1$.
(c) Is $h(x)$ a 1-1 function? why or why not?

33. (8 points) A wire of length 12 feet is cut into two pieces and each piece is bent into a square. Let $x$ be the length of the side of one square and $y$ be the length of the side of the other square. Write the sum of the areas of the two squares as a function of $x$.

34. (12 points) On the axes below, sketch the graph of a function $f(x)$ that satisfies all the following properties:

- $f(x) = \frac{1}{x}$ on $(1, \infty)$
- $f(x)$ is increasing on $(-4, -2) \cup (-2, 0)$
- $f(x)$ is decreasing on $(0, 1) \cup (1, \infty)$
- $f(x)$ is negative on $[-4, -3) \cup (-2, 1]$,
- $f(x)$ is positive on $(-3, -2) \cup (1, \infty)$
- the domain of $f(x)$ is $[-4, -2) \cup (-2, \infty)$

**Note:** There are many possible correct answers. After you finish, go back and make sure your function satisfies all the listed properties. If you make any changes, go back and check the other conditions again.

Show all your work!

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1Your friends in Math 10a are, right now, being asked to figure out what value of $x$ will minimize the sum of the two areas.
35. (15 points) Use transformations to graph the function \( f(\theta) = 2\cos(\theta) + 1 \), on the grid below. Make sure your graph extends from \(-2\pi\) to \(2\pi\). Then answer the following questions.

(a) Find the range of \( f \).

(b) On what interval(s) in \((0, 2\pi)\) is the function increasing?

(c) On what interval(s) in \([0, 2\pi]\) is the function negative?

Hint: It may help to solve the equation \( f(\theta) = 0 \).

36. (6 points) Determine whether each of the following two statements is true or false.

You must justify your answer.

Note: In mathematics, “true” means that the statement must always be true. “False” means that the statement may sometime be false.

(a) True or False: If \( \theta \) and \( \phi \) are coterminal \(^2\) angles then \( \cos(\theta) = \cos(\phi) \).

(b) True or False: Given two angles \( \theta \) and \( \phi \), if \( \cos(\theta) = \cos(\phi) \), then \( \theta \) and \( \phi \) must be coterminal.

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\(^2\)Recall: Two angles in standard position are “coterminal” if they have the same terminal (ending) side.
37. Simplify the following:

(a) \( \frac{(a^3b^{-3}c^3)^{-2}}{(a^{-1}b^{-3}c^{-5})^3} \)

(b) \( 8\sqrt{2} - 6\sqrt{20} - 5\sqrt{8} \)

38. Convert the following to exponential notation and simplify. Do not convert back to radical notation.

(a) \( \sqrt[3]{3x^2y} \sqrt[3]{9xy^4} \)

(b) \( \sqrt[3]{x^6} \)

39. Evaluate \( (-8)^{-\frac{3}{4}} \).

40. Give an example of a polynomial that satisfies all the following conditions: (1) the polynomial has degree 3, (2) the constant term is negative, and (3) the coefficient of the term of degree 2 is irrational.

41. Multiply and simplify: \( (\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})(x - y) \)

42. Factor the following completely:

(a) \( 6y^4 - 96x^4 \)

(b) \( 12x^2 + x^4 - 64 \)

43. Simplify the following:

(a) \( \frac{y^2 - 9}{y^2 - 49} \div \frac{y^2 - 8y + 15}{y^2 + 5y - 14} \)

(b) \( 1 + \frac{x}{x - 3} - \frac{2x^2}{x^2 - 9} \)

44. Solve the following. Write your answers in (d)–(f) in interval notation.

(a) \( x = x^2 - 1 \)

(b) \( x^4 = 6x^2 \)

(c) \( \frac{x^2 - 2}{x - 1} = 0 \)

(d) \( x^3 + 3x^2 - 28x < 0 \)

(e) \( \frac{1}{x} - \frac{1}{x + 3} \geq 0 \)

(f) \( \frac{x^2e^x - 5e^x}{x + 3} \leq 0 \)

45. A line passes through the points \((6, 1)\) and \((-2, 5)\).

(a) Find the equation of the line in point-slope form. Graph the line.

(b) Find the equation of the line that passes through the point \((1, 1)\) and is parallel to the line you just found. Graph it on the same axes with your first line.

46. Find the equation of the line that passes through the point \((-3, 2)\) and is perpendicular to the line \( y = 7 \).

47. The pressure \( p \) acting at a point in a liquid is directly proportional to the distance \( d \) from the point to the surface of the liquid. Suppose that in a certain oil tank the pressure at a depth of 2 feet from the surface is 118 lb/in\(^3\). Find the pressure at a depth of 5 feet.
48. Consider the relationship given by the following table:

<table>
<thead>
<tr>
<th>Day of the week</th>
<th>Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>Partly cloudy</td>
</tr>
<tr>
<td>Tuesday</td>
<td>Sunny</td>
</tr>
<tr>
<td>Wednesday</td>
<td>Rainy</td>
</tr>
<tr>
<td>Thursday</td>
<td>Sunny</td>
</tr>
<tr>
<td>Friday</td>
<td>Foggy</td>
</tr>
</tbody>
</table>

Is the day of the week a function of the forecast? Why or why not? Be very specific.

49. In each of the following, find \( \frac{f(x + h) - f(x)}{h} \), and simplify it.
   
   (a) \( f(x) = \frac{2}{x + 1} \)  
   (b) \( f(x) = 3x^2 - 5x + 1 \)

50. Find the domain of each of the following functions:
   
   (a) \( f(x) = \frac{1}{x^2e^x - 3e^x} \)  
   (b) \( f(x) = \ln(x^2 - 5x) \)

51. Graph each of the following:
   
   (a) \( f(x) = \begin{cases} 
                           2^{-x}, & \text{if } x \leq -1 \\
                           x^3, & \text{if } -1 < x < 1 \\
                           \frac{1}{x^2}, & \text{if } x \geq 1 
                     \end{cases} \)  
   (b) \( g(x) = \begin{cases} 
                       \frac{1}{x + 1}, & \text{if } x < 0 \\
                       -|x - 1|, & \text{if } 0 \leq x < 2 \\
                       \sqrt{x - 2}, & \text{if } x \geq 2 
                       \end{cases} \)

   Then answer the following questions about the function \( f(x) \) that you got in part (a):
   
   (i) On what interval(s) is \( f(x) \) positive? On what interval(s) is \( f(x) \) negative?
   (ii) On what interval(s) is \( f(x) \) increasing? On what interval(s) is \( f(x) \) decreasing?

52. Do problems #5 and 12 on page 177, and problem #64 on page 210.

53. A rancher has 20 miles of fencing to fence a rectangular piece of grazing land along a straight river. If no fence is required along the river, and the sides perpendicular to the river are \( x \) miles long, express the area of the rectangle as a function of \( x \). (You do not have to simplify your answer.)

54. Suppose that the same rancher decided to use a heavy-duty fencing for the sides perpendicular to the river; this fencing cost $120 per mile. Because the horses who will graze in the enclosed rectangle will tend to stay near the river, the rancher decides that the side opposite the river can be fenced with a cheaper fencing which costs $80 per mile. Express the total cost of fencing in the rectangle as a function of \( x \). (You do not have to simplify your answer.)

55. A soft drink company sells its product in a cylindrical tin can with volume 100 cubic centimeters. Express the surface area of the can as a function of the radius of the base. (You do not have to simplify your answer.)
56. A shipping company plans to manufacture a box with a square base and a volume 220 cubic feet. The base of the box will be made of heavy-duty sheet metal that costs $6.00 per square foot. The sides and top of the box will be made of a somewhat lighter metal that costs $4.50 per square foot. Express the cost of manufacturing the box as a function of the length of the side of the base. (You do not have to simplify your answer.)

57. A boat spends the night anchored 10 miles directly east of Rockport Harbor. At 8 am, the boat weighs anchor and begins travelling due east at a speed of 45 miles per hour. Write a function which expresses the distance of the boat from Rockport Harbor t hours after it weighs anchor. If the boat stops travelling at 3 pm that afternoon, what is the domain of the function? Write the domain in interval notation.

58. As a diver descends into the ocean, the pressure p increases with depth d, and p is a linear function of d. The pressure is 15 psi at the surface, whereas the pressure 33 feet below the surface. Find the function that expresses p as a function of d.

59. Consider the following four functions:
   
   \[ f(x) = x + 1, \quad g(x) = \frac{e^x}{x^2 - 1}, \quad h(x) = \sqrt[3]{x^3 - 8}, \quad k(x) = 2\ln x + 3. \]

   (a) Find the following:
      
      (i) \((gh)(0)\)
      (ii) \(\left(\frac{f}{k}\right)(1)\)
      (iii) \((f \circ h)(0)\)
      (iv) \((k \circ g)(\sqrt{2})\)
      (v) \((h \circ h)(2)\)

   (b) Find the x and y-intercept(s) (if any) of \(f(x), g(x),\) and \(k(x)\).

   (c) Find \((k \circ f)^{-1}(x)\).

60. In each of the following, find \(f^{-1}(x)\).

   (a) \(f(x) = 4 + \sqrt{x - 1}\)
   (b) \(f(x) = e^{2x + 1}\)

61. Let \(f(x) = \begin{cases} 
3x - 3, & \text{if } x < 1 \\
2\ln x, & \text{if } x \geq 1 
\end{cases}\). Sketch the graph of \(f(x)\). Is \(f(x)\) one-to-one? If so, sketch the graph of \(f^{-1}(x)\).

62. Let \(f(x) = x^3 + 5x - 2\). Find \((f \circ f^{-1})(2)\) and \((f^{-1} \circ f)(-4)\).

63. Evaluate each of the following:

   (a) \(\log_3 \left(\frac{9}{7}\right)\)
   (b) \(\log_{\sqrt{2}} 8\)
   (c) \(\ln \sqrt{e} - e^{\ln \frac{1}{7}}\)

64. Write the following as a single logarithm: \(4\ln x - \frac{1}{2}\ln x + 3\ln y\).

65. Do problems #9 on page 313, problems #48, 53 and 60 on page 323, problem #70 on page 330 and problem #85 on page 339. In #85b on page 339, use the fact that \(e^{-\frac{7}{2}} \approx .5647\) to get a decimal approximation for the answer.

66. Suppose that the graph of an exponential function \(f(x) = Ca^x\) passes through the points \((0, 3)\) and \((2, 12)\). Find the values of \(C\) and \(a\).
67. Solve the following. Write your answers in (d), (g) and (h) in interval notation.

(a) \( \frac{2^x}{x - 7} = 0 \)  
(b) \( e^{2x} - 2e^x - 15 = 0 \)  
(c) \( \ln x - \ln(x + 1) = \ln 3 \)  
(d) \( 2x \ln x - x \leq 0 \)  
(e) \( \log_7 x^2 = 2 \)  
(f) \( \ln 3 + 2 \ln x = \ln(2 - x) \)  
(g) \( (\ln x)^2 - 3 \ln x < 0 \)  
(h) \( \frac{2 - \ln x}{x - e^x} \leq 0 \)

68. For each of the following functions \( h \), find functions \( f \) and \( g \) such that \( h = f \circ g \). (You may not use \( f(x) = x \) or \( g(x) = x \).)

(a) \( h(x) = \sin(x^2 - 2x + 3) \)  
(b) \( h(x) = \sqrt{\ln x + 3} + 5 \)  
(c) \( h(x) = \cos x \cdot x \)

69. In the triangle \( \triangle ABC \) shown below, suppose that we know that \( AB = 8 \) and cot \( \theta = 2 \). Find \( AC \).

![Triangle ABC](image)

70. Find two angles, one with positive measure and one with negative measure, that are coterminal with \( \theta = \frac{2\pi}{9} \) radians.

71. Convert from degrees to radians or vice versa:

(a) 200\(^\circ\)  
(b) \( \frac{\pi}{10} \) radians  
(c) \( -\frac{4\pi}{5} \) radians

72. Find the following (without using a calculator):

(a) \( \sin 270^\circ \)  
(b) \( \cos 120^\circ \)  
(c) \( \tan(-240^\circ) \)  
(d) \( \csc 450^\circ \)  
(e) \( \cot 0^\circ \)  
(f) \( \sec \frac{7\pi}{6} \)  
(g) \( \sin \frac{5\pi}{3} \)  
(h) \( \cot \frac{13\pi}{6} \)  
(i) \( \tan \frac{5\pi}{4} \)  
(j) \( \cos(-2\pi) \)

73. Suppose that \( \theta \) is an angle, and that \( \sec \theta = 5 \) and \( \tan \theta < 0 \). Find the following:

(a) \( \cos \theta \)  
(b) \( \cot \theta \)  
(c) \( \csc \theta \)

74. The terminal side of an angle \( \theta \) intersects the unit circle at a point \( P \), and the coordinates of \( P \) are \((\sqrt{2}k, \sqrt{7}k)\). Suppose you know that \( P \) is in the third quadrant.

(a) Find \( k \).  
(b) Find sin \( \theta \).  
(c) Find tan \( \theta \).

75. In each of the following, find the values of \( \theta \) in \([0, 2\pi]\) which make the equation true.

(a) \( \cos \theta = -\frac{\sqrt{3}}{2} \)  
(b) \( \tan \theta = -1 \)  
(c) \( \cos^2 \theta = 1 \)  
(d) \( \csc^2 \theta = 4 \)

76. Is it possible to find an angle \( \theta \) such that \( \cos \theta = 2 \)? Why or why not?
77. Let $f(x) = \sin(2x)$ and let $g(x) = \cos(x + \frac{\pi}{2})$. Find the following (if they exist):

(a) $f\left(\frac{\pi}{4}\right)$  
(b) $g\left(\frac{\pi}{4}\right)$  
(c) $f\left(\frac{2\pi}{3}\right)$  
(d) $(f \circ g)(-\pi)$  
(e) $(g \circ f)(x)$  
(f) $(g \circ f)(0)$

78. Sketch the graph of $f(x) = 2\sin(x - \pi)$.

79. Sketch the graph of the following function: $f(x) = \begin{cases} \cos x, & \text{if } x \leq 0 \\ -2\sin x, & \text{if } x > 0. \end{cases}$

80. The position of a weighted spring moving up and down along a vertical coordinate line is given by the function $f(t) = 2e^{-t}\sin t$. When $f(t) = 0$ the weight is at its equilibrium position. We take the downward direction to be positive.

(a) How far is the spring from its equilibrium position at time $t = \frac{\pi}{6}$ seconds? Is it above or below its equilibrium position?

(b) Find the average velocity of the spring between time $t = \frac{\pi}{2}$ seconds and time $t = \pi$ seconds.

(c) When does the weight pass through its equilibrium position for the second time?