Tidying up our prev. discussion on analytic techniques

In the real world, people want more than "the solution is exp decay with an undetermined const."

They want to know the answer to "how many fish will there be next year?"

Initial-value problems

$$\begin{cases} \dfrac{dy}{dt} = f(t, y) \\ y(t_0) = y_0 \quad \text{initial cond} \end{cases}$$

e.g.

$$\begin{cases} \dfrac{dy}{dt} = 12t^3 - 2\sin t \\ y(0) = 3 \end{cases}$$

$\begin{cases} y(0)=3 \end{cases}$

$$\int dy = \int (12t^3 - 2\sin t)\,dt$$

$$\rightarrow \quad y(t) = 3t^4 + 2\cos t + C \qquad \text{general soln}$$

Use initial cond to determine C

$$y(0)=3 \rightarrow \quad 2 + C = 3 \qquad C = 1$$

so $\underline{y(t) = 3t^4 + 2\cos t + 1}$ soln to IVP

(ex) Deposit \$5,000 in account with 2% int. rate & cts compounding. How much \$ in 10 yrs?

Let $A(t)$ = account value @ time t

$$\frac{dA}{dt} = rA = 0.02A \qquad \text{(cts compounding) eqn}$$

Separable eqn: $\int \frac{dA}{A} = \int 0.02\,dt$

separable eq.

$$\ln|A| = 0.02t + C$$

$$A(t) = Ce^{0.02t}$$

$$A(0) = 5{,}000 \Rightarrow C = 5{,}000$$

$$A(t) = 5{,}000e^{0.02t}$$

$$t = 10 \Rightarrow A(10) = 5{,}000e^{0.2} \approx 6{,}107$$

(No compounding: 6,000)

$$t = 40 \Rightarrow A(40) = 11{,}127$$

(No compounding: 9,000)

2 Pitfall of sep. eqns technique

$$\frac{dy}{dt} = y^2$$

$$\Rightarrow \int \frac{dy}{y^2} = \int dt \Rightarrow \frac{-1}{y} = t + C$$

$$y(t) = -\frac{1}{t+C}$$

Is this the general soln?

~~How~~ about $y(t) \equiv 0$?

When dividing, we implicitly assumed $y \neq 0$

$$\therefore \quad \begin{cases} y(t) = \dfrac{-1}{t+c} \\ y(t) \equiv 0 \end{cases}$$ is more "general" soln

What is really happening in sep. eqn. technique ?

$$\frac{dy}{dt} = f(t,y) = g(t)h(y)$$

$$\Rightarrow \frac{1}{\quad} \frac{dy}{\quad} = g(t)$$

$$h(y)\,dt \quad = g(t)$$

Really $\dfrac{1}{h(y(t))}\dfrac{dy}{dt} = g(t)$

Integrate both sides

Now, u-substitution