Tidying up an prev. discussion on analytic techniques.

In the real world, people want more than “the solution is exp decay with an undetermined const.”

They want to know the answer to “how many fish will there be next year?”

Initial-value problems

\[
\begin{align*}
\frac{dy}{dt} &= f(t, y) \\
y(t_0) &= y_0, \text{ initial cond}
\end{align*}
\]

E.g.,

\[
\begin{align*}
\frac{dy}{dt} &= 12t^3 - 2 \sin t \\
y(0) &= 3
\end{align*}
\]
\[ y(0) = 3 \]

\[ \int dy = \int (12t^2 - 2\sin t) \, dt \]

\[ y(t) = 3t^4 + 2\cos t + C \quad \text{general soln} \]

Use initial cond to determine \( C \)

\[ y(0) = 3 \Rightarrow 2 + C = 3 \quad C = 1 \]

\[ y(t) = 3t^4 + 2\cos t + 1 \quad \text{solln to IVP} \]

\[ \text{(ex)} \quad \text{Deposit } \$5000 \text{ in account with } 2\% \text{ int. rate } \Rightarrow \text{cts compounding} \]

How much \( \$ \) in 10 yrs?

Let \( A(t) = \text{account value @ time } t \)

\( \frac{dA}{dt} = rA \) \((\text{cts compounding})\)

\( r = 0.02 \)

Solve w.r.t. \( A \):

\[ \int \frac{dA}{A} = \int 0.02 \, dt \]
\ln |A(t)| = 0.02t + C
\Rightarrow A(t) = Ce^{0.02t}

\begin{align*}
A(0) & = 5,000 \Rightarrow C = 5,000 \\
A(t) & = 5,000 e^{0.02t} \\

t = 10 & \Rightarrow A(10) = 5,000 e^{0.2} \approx 6,107 \\
(No \; compounding: \; 6,000) \\
\end{align*}

\begin{align*}
\frac{dy}{dt} & = y^2 \\
\Rightarrow y + 1 & = t + C \\
\end{align*}

Pitfall of separate equation technique
\[ \int \frac{dy}{y^2} = \int dt \quad \Rightarrow \quad \frac{-1}{y} = t + C \]

\[ y(t) = -\frac{1}{t + C} \]

do this the general soln?

How about \( y(t) = 0 \)?

When dividing, we implicitly assumed \( y \neq 0 \)

\[ \therefore \quad \left\{ \begin{array}{l} y(t) = \frac{-1}{t + C} \\
y(t) = 0 \end{array} \right. \]

to "general" soln

What is really happening in sep. eqn. technique?

\[ \frac{dy}{dt} = f(t, y) = g(t)h(y) \]

\[ \Rightarrow \quad \frac{1}{h(y)} \, dy = g(t) \, dt \]
Really

Integrate both sides

Now, \( h(y') \, dt = g(t) \)

\[
\int h(y(t)) \, dt = \int g(t) \, dt
\]