Nonlinear systems

So far only considered eqn of a single function

We can use systems of diff eqns to describe dynamics of multiple coupled fns

Consider a model of the interaction b/w populations of predators (foxes) and prey (rabbits)

Let $R(t)$ - size of rabbit pop @ time t

$F(t)$ - size of fox pop @ time t

Assume:

- If no foxes present, rabbits grow exponentially

- Foxes eat rabbits, at rate proportional to their interaction rate. Interaction rate $= kRF$

- Without rabbits, fox pop. decreases exponentially

This leads to model

$$\frac{dR}{dt} = \alpha R - \beta RF$$

$$\frac{dF}{dt} = \delta RF - \gamma F$$

Where α = rabbit growth rate

γ = fox death rate

β = prop. const. for rabbit death

δ = prop. const for fox birth

The qualitative ideas & maths we've discussed can be generalized to systems of eqns

In-class exercise

Fix $\alpha = \beta = \gamma = \delta = 1$ &

1) Find steady-state solns.

2) Can you determine stability?


stable
limit
cycles